Gravitational lensing constraint on the cosmic equation of state

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Abstract

Recent redshift-distance measurements of Type Ia supernovae at cosmological distances suggest that two-third of the energy density of the universe is dominated by dark energy component with an effective negative pressure. This dark energy component is described by the equation of state $p_x = w\rho_x$ ($w \geq -1$). We use gravitational lensing statistics to constrain the equation of state of this dark energy. We use $n(\Delta\theta)$, image separation distribution function of lensed quasars, as a tool to probe w. We find that for the observed range of $\Omega_m \sim 0.2 - 0.4$, w should lie between $-0.75 \leq w \leq -0.42$ in order to have five lensed quasars in a sample of 867 optical quasars. This limit is highly sensitive to lens and Schechter parameters and evolution of galaxies.

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Recent redshift-distance measurements of Type Ia supernovae at cosmological distances suggest that 2/3 of the energy density of the universe could be in the form of a dark energy component with an effective negative pressure [1, 2, 3, 4]. This causes the universe to accelerate. There is other observational evidence which also support the existence of an unknown component of energy density with pressure $p = w\rho$. This includes recent measurements of the angular power spectrum which peak at $l \sim 200$ [5] and dynamical estimates of the matter density $\Omega_m = 0.35 \pm 0.07$ [6]. Many candidates have been proposed for this

dark energy component which is characterised by an equation of state $-1 \leq w \equiv p/\rho \leq 0$.

The first candidate is the cosmological constant characterised by $\omega = -1$. In this case a vacuum energy density or $\Lambda -$ term ($8\pi \mathrm{G}\rho_{\mathrm{vac}}$) is independent of time (For a recent review see [7]). There are several other possibilities for this dark energy:

1. A varying $\Lambda -$ term[8]
2. Rolling scalar fields (quintessence)[9]
3. Frustrated network of topological defects in which $w = -N/3$ where N is dimension of the defect [10].
4. X-matter [11]

There are some independent methods from which the limit on w can be constrained. For example Perlmutter et al. (1999) constrained $w < -0.6$ (95 % CL) using large-scale structure and SNe Ia in a flat universe [2]. Waga & Miceli (1999) used lensing statistics and supernovae data to constrain w and found that $w < -0.7$(68 % CL) in a flat universe [12]. Recently Lima & Alcaniz (2000) used age measurements of old high redshifts galaxies (OHRG) to limit w [13]. For $\Omega_m = 0.3$, the ages of OHRG give $w \leq -0.27$. By combining "cosmic concordance" method with maximum likelihood estimator, Wang et al. (2000) have found that the best-fit model lies in the range $\Omega_m = 0.33 \pm 0.05$ with an effective equation-of-state $w \sim -0.6 \pm 0.07$ [14].

which is characterized by its equation of state ,$p_x = w_x\ \rho_x$ and $w_x \geq -1$.We use gravitational lensing as a tool to constrain w for X-matter. In section 2 we describe some formulae (age-redshift relationship and angular diameter distance) used in lensing statistics. In section 3 we explain the use of $n(\Delta\theta)$, the image separation distribution function of lensed quasars, as a tool to constrain the cosmic equation of state of dark energy. We briefly summarize our results in sec. 4.

2 Cosmology with dark energy

We consider spatially flat, homogeneous and isotropic cosmologies. The Einstein equations in the presence of nonrelativistic matter and dark energy are given by: For more details see ref.[15]

$$H^2(z) \equiv (\frac{\dot{R}}{R})^2 = H_0^2 \left[\Omega_m \, (\frac{R_0}{R})^3 + \Omega_x \, (\frac{R_0}{R})^{3(1+\omega)}\right] \tag{1}$$

$$(\frac{\ddot{R}}{R}) = \frac{-1}{2} H_0^2 \left[\Omega_m \, (\frac{R_0}{R})^3 + (1+3\omega) \;\; \Omega_x \, (\frac{R_0}{R})^{3(1+\omega)}\right] \tag{2}$$

where the dot represents derivative with respect to the time.

$$\Omega_m = \frac{8\,\pi\,G}{3H_0^2}\rho_{m0} \;\text{ and }\; \Omega_x = \frac{8\,\pi\,G}{3H_0^2}\rho_{x0}$$

where H_0 is the Hubble constant ρ_{m0} and ρ_{x0} are the nonrelativistic matter density and the dark energy density respectively.

The age-redshift relationship: The age of the universe at the redshift z is given by

$$H_0\, t(z) = H_0 \int_z^\infty \frac{dz'}{(1+z')H(z')}$$

$$= \int_z^\infty \frac{dz'}{(1+z')\sqrt{\Omega_m(1+z')^3 + \Omega_x(1+z')^{3(1+w)}}} \tag{3}$$

The angular diameter distance

$$d_A(z_1, z_2) = \frac{R_0}{(1+z_2)} \int_{z_1}^{z_2} \frac{dz}{\sqrt{\Omega_m(1+z)^3 + \Omega_x(1+z)^{3(1+w)}}} \tag{4}$$

$n(\Delta\theta)$ is the image separation distribution function for lensed quasars. To understand $n(\Delta\theta)$ we have to find the optical depth. The lensing probability or the optical depth $d\tau$ of a beam encountering a lensing galaxy at redshift z_L in traversing the path of dz_L is given by a ratio of differential light travel distance cdt to its mean free path between successive encounters with galaxies $1/n_L(z)\sigma$, where $n_L(z)$ is the number density of galaxies and σ is effective cross-section for strong lensing events. Therefore

$$d\tau = n_L(z)\sigma \frac{cdt}{dz_L} dz_L, \tag{5}$$

We assume $n_L(z)$ is the conserved comoving number density of galaxies (lenses), $n_L(z) = n_0(1+z_L)^3$. The present-day galaxy luminosity function can be described by the Schechter function [16]

$$\Phi(L, z=0)dL \;=\; \phi_* \left(\frac{L}{L_*}\right)^{\alpha} \; exp\left(-\frac{L}{L_*}\right) \frac{dL}{L_*} \tag{6}$$

$$n_0 = \int_0^{\infty} \Phi(L)dL \tag{7}$$

The Singular Isothermal Sphere (SIS) with one dimensional velocity dispersion v is a good approximation to account for the lensing properties of a real galaxy. The deflection angle for all impact parameters is given by $\hat{\alpha} = 4\pi v^2/c^2$. The lens produces two images if the angular position of the source is less than the critical angle β_{cr}, which is the deflection of a beam passing at any radius through an SIS:

$$\beta_{cr} = \hat{\alpha} D_{LS}/D_{OS}, \tag{8}$$

We use the notation $D_{OL} = d(0, z_L)$, $D_{LS} = d_A(z_L, z_S)$, $D_{OS} = d_A(0, z_S)$, where $d_A(z_1, z_2)$ is the angular diameter distance between the redshift z_1 and z_2. Then the critical impact parameter is defined by $a_{cr} = D_{OL}\beta_{cr}$ and the cross-section is given by

$$\sigma = \pi a_{cr}^2 = 16\pi^3 \left(\frac{v}{c}\right)^4 \left(\frac{D_{OL}D_{LS}}{D_{OS}}\right)^2, \tag{9}$$

The differential probability $d\tau$ of a lensing event can be written as

$$\frac{d\tau}{dz_L} = n_L(z) \left[\frac{16\pi^3}{cH_0^3} v^4 \left(\frac{D_{OL} D_{LS}}{R_0 D_{OS}} \right)^2 \frac{1}{R_0} \right] \frac{cdt}{dz_L} dz_L, \tag{10}$$

The total optical depth can be obtained by integrating $d\tau$ from 0 to z_S which is equal to [17]

$$\tau(z_S) = \frac{F^*}{30} (D_{OS}(1+z_S))^3 (R_0)^{-3} \tag{11}$$

where

$$F^* = \frac{16\pi^3}{cH_0^3} \phi_* \, v_*^4 \Gamma \left(\alpha + \frac{4}{\gamma} + 1 \right)$$

We neglect the contribution of spirals as lenses as their dispersion velocity is small as compared to ellipticals. The relationship between the luminosity and velocity is given by the Faber-Jackson relationship $\frac{L}{L_*} = \left(\frac{v}{v_*} \right)^\gamma$

The list of Schechter and Lens parameters for E/SO galaxies that we use are the following [18]

$Survey$	α	γ	$v^*(Km/s)$	$\phi^*(Mpc^{-3})$	F^*
$LPEM$	+0.2	4.0	205	3.2×10^{-3}	0.010

The differential optical depth of lensing in traversing dz_L with angular separation between ϕ and $\phi + d\phi$, is given by [19]

$$\begin{aligned} \frac{d^2\tau}{dz_L d\phi} d\phi \, dz_L &= \frac{\gamma/2}{\phi} \left[\frac{D_{OS}}{D_{LS}} \phi \right]^{\frac{\gamma}{2}(\alpha+1+\frac{4}{\gamma})} exp \left[- \left(\frac{D_{OS}}{D_{LS}} \phi \right)^{\frac{\gamma}{2}} \right] \frac{cdt}{dz_L} \\ & \times F^* \frac{(1+z_L)^3}{\Gamma\left(\alpha + \frac{4}{\gamma} + 1\right)} \left[\left(\frac{D_{OL} D_{LS}}{R_0 D_{OS}} \right)^2 \frac{1}{R_0} \right] d\phi \, dz_L \end{aligned} \tag{12}$$

The normalized image angular separation distribution for a source at z_S is obtained by integrating the above expression over z_L

$$\frac{d\mathcal{P}}{d\phi} = \frac{1}{\tau(z_S)} \int_0^{z_s} \frac{d^2\tau}{dz_L d\phi} dz_L \tag{13}$$

We include two correction factors in the probability of lensing: (1) magnification bias (2) selection function due to finite resolution and dynamic range.

quasar is lensed. The bias for a quasar at redshift z with apparent magnitude m is written as

$$B(m, A_1, A_2, z) = (\frac{dN}{dm})^{-1} \int_{A_1}^{A_2} \frac{dN}{dm}(m + 2.5\, logA, z)\; p(A) dA \tag{14}$$

where $p(A)$ is the probability distribution for a greater amplification A which is $8/A^3$ for SIS model. We use $A_1 = 2$ and $A_2 = 10^4$. We use the quasar luminosity function as given by Kochanek [17].

$$\frac{dN}{dm} \propto \left(10^{-x(m-m_0(z))} + 10^{-y(m-m_0(z))}\right)^{-1} \tag{15}$$

where

$$\mathrm{m}_0\,(z) = \begin{cases} m_0 + (z+1) & \text{if} \quad z < 1 \\ m_0 & \text{if} \quad 1 < z < 3 \\ m_0 - 0.7(z-3) & \text{if} \quad z > 3. \end{cases} \tag{16}$$

We use $x = 1.07$, $y = 0.27$ and $m_0 = 18.92$. We considered a total of 862 ($z > 1$) highly luminous optical quasars plus five lenses.

Selection effects are caused by limitations on dynamic range, limitations on resolution and presence of confusing sources such as stars. Therefore we must include a selection function to correct the probabilities. In the SIS model the selection function is modeled by the maximum magnitude difference $\Delta m(\theta)$ that can be detected for wo images separated by $\Delta\theta$. This is equivalent to a limit on the flux ratio ($f > 1$) between two images $f = 10^{0.4\,\Delta m(\theta)}$. The total magnification of images becomes $A_f = A_0(f+1)/(f-1)$. So the survey can only detect lenses with magnifications larger than A_f. This sets the lower limit on the magnification. Therefore the A_1 in the bias function gets replaced by $A_f(\theta)$. To get selection function corrected probabilities, we divide our sample into two parts: the ground based surveys and the HST Snapshot survey. We use the selection function as suggested by Kochanek [20]. The corrected image separation distribution function for a single source at redshift z_S is given as [17, 21]

$$\times F^* \frac{cdt}{dz_L} \frac{(1+z_L)^3}{\Gamma\left(\alpha + \frac{4}{\gamma} + 1\right)} \left[\left(\frac{D_{OL} D_{LS}}{R_0 D_{OS}} \right)^2 \frac{1}{R_0} \right] \, dz_L \tag{17}$$

Similarly the corrected lensing probability for a given source at redshift z is given as [17, 21]

$$P = 4 \int \frac{d\mathcal{P}}{d\phi} B(m, A_f(\theta), A_2, z) \, d\phi \tag{18}$$

Here ϕ and $\Delta\theta$ are linked through $\phi = \frac{\Delta\theta}{8\pi(v^*/c)^2}$.
We sum the lensing probabilities P_i for the individual QSOs in order to get the number of lensed quasars, $n = \sum P_i$. Similarly for the image-separation distribution function $n(\Delta\theta) = \sum P_i(\Delta\theta)$. The summation sign is over all quasars in a given sample.

4 Results and Discussions

Gravitational lensing statistics is a sensitive cosmological probe for determination of the nature of dark energy. This is because statistics of multiply imaged lensed quasars can probe the universe to a redshift $z \sim 1$ or even higher. This is the time when dark energy starts playing a dominant role in the dynamics of universe.

That lensing statistics can be used as a tool to constrain various dark energy candidates has been known for some time. Kochanek (1996) gave a 2σ upper bound on $\Omega_\Lambda < 0.66$ from multiple images of lensed quasars[17]. Waga & Miceli (1999) used the combined analysis of gravitational lensing and Type Ia supernovae to constrain the time dependent cosmological term $\Lambda \propto R^{-m}$ [12]. This combined analysis shows that $w \leq -0.7$. Cooray and Huterer (1999) also used lensing statistics to constrain various quintessence models [22].

Wang et al.(2000) raise several points while using gravitational lensing statistics as a tool to probe cosmology [14]. We try to include some of the points in this work. First, the role of spirals as lenses are completely neglected as their dispersion velocity is small as compared to ellipticals. Secondly, the sation ϕ_* for E/SO galaxies. Earlier work on lensing statistics used $\alpha \sim -1$, which implies the existence of numerous faint E/SO galaxies acting as lenses. Because of limited resolution, this faint part of the luminosity function is still uncertain. Moreover these parameters have been determined in a highly correlated manner in a galaxy survey which has not been taken care most of the time in earlier work on lensing statistics. Therefore to combine the parameters from various surveys will cause the errors. We use the updated luminosity function of LPEM. The LPEM luminosity function is characterised by the shallow slope α at faint end and the smaller normalisation ϕ_* which shifts the distribution to large image separations.

We use the image separation distribution function function $n(\Delta\theta)$, to constrain the cosmic equation of state for the dark energy. $n(\Delta\theta)$ depends upon w through the angular diameter distances as shown in Section 2. By varying w, the distribution function changes which on comparison with the observations gives constraint on w.

Fig.1 shows the expected number of lensed quasars $n(\Delta\theta \leq 4'')$ as a function of Ω_m in a flat universe with $w = -1$. Comparing the predicted numbers with the observed five lenses, the best value of Ω_m is 0.45. Therefore the gravitational lenses do not favour a large cosmological constant. This result is however, sensitive to Lens and Schechter parameters.

In Fig.2 $n(\Delta\theta)$ is plotted against $\Delta\theta$ (image separation) in the flat cosmology for various values of w. The plotted rectangles indicate the image-separation distribution of the five lensed quasars in the optical sample considered in this calculation. As indicated by recent distance measurements of Type Ia supernovae, we fix $\Omega_m = 0.3$ in Fig. 2. On comparing the theoretical prediction of image distribution function with the observations we see that $w \sim -0.53$ reproduces the data better than the other values of w. $w = -1$ produces a large number of lenses which don't match with the observations. Hence it is ruled out for $\Omega_m = 0.3$. Similarly $w = 0$ will give less number of lensed quasars which again don't match with observations. If we increase the value of Ω_m in a flat universe, a smaller value of w is required to match with observations. The magnitude of the peak $n(\Delta\theta)$ is sensitive to the value of Ω_m in the flat universe. Larger the value of Ω_m for a fixed value of z, smaller position of the peak of $n(\Delta\theta)$ is sensitive to the value α i.e faint end slope of luminosity function. If we take the conventional value of $\alpha = -1$, the peak will shift to a lower value of $\Delta\theta$ or in other words it will predict lensed images with small angular separations[23].

The difficulty is that the number of observed lensed QSOs in this sample is too small to put strong constraints on w. Extended surveys are required to establish $n(\Delta\theta)$ as a powerful tool. The upcoming Sloan Digital Sky Survey which is going down to $1-\sigma$ magnitude limit of ~ 23 will definitely increase our undestanding on lensing phenomena and on cosmological parameters.

In Fig 3. we show the $\Omega_m - w$ plane. For a fixed value of five observed lensed quasars, each point on the curve corresponds to the pair of (Ω_m, w). If the matter contribution Ω_m increases, smaller value of w is required to observe five lenses. The dotted lines in the Fig. corresponds to the observed range $\Omega_m \sim 0.2 - 0.4$ [24]. The corresponding range for w in the flat universe is $-0.75 \leq w \leq -0.42$. These results agree very well with the limit obtained on w by other independent methods as described in the introduction. With this given range of w we can rule out two dark energy candidates. First , cosmic strings ($w \sim -0.33$) and second, cosmological constant ($w = -1.0$). It is quite interesting to compare this result with limit obtained on w by using the MAXIMA-1 AND BOOMERANG-98 treating the dark energy as a quintessence, $-1 \leq w \leq -0.5$. Our constraint on w is much tighter than obtained by using the MAXIMA-1 AND BOOMERANG-98 [25]. But the strength of the constraints obtained here depends strongly on lens and Schechter parameters, evolution of galaxies and of course on the quality of the lensing data. So we need a larger sample of high-z QSOs, better understanding of the formation and evolution of galaxies over wide range of redshifts and right galaxy distribution parameters before any definitive and strong statements can be made regarding the constraints on w by gravitational lensing.

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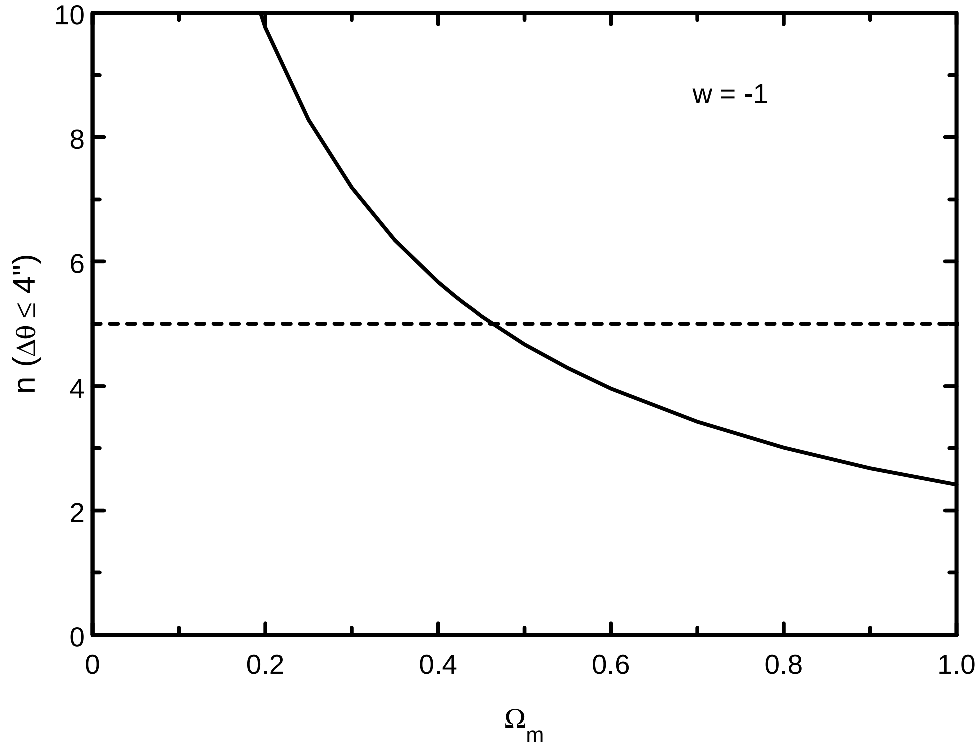

Figure 1: Predicted total number of lenses with $\Delta\theta \leq 4''$ as function of Ω_m for flat universe with $w = -1$ (constant Λ). The observed five lensed QSOs from optical lens survey is shown by dotted line.

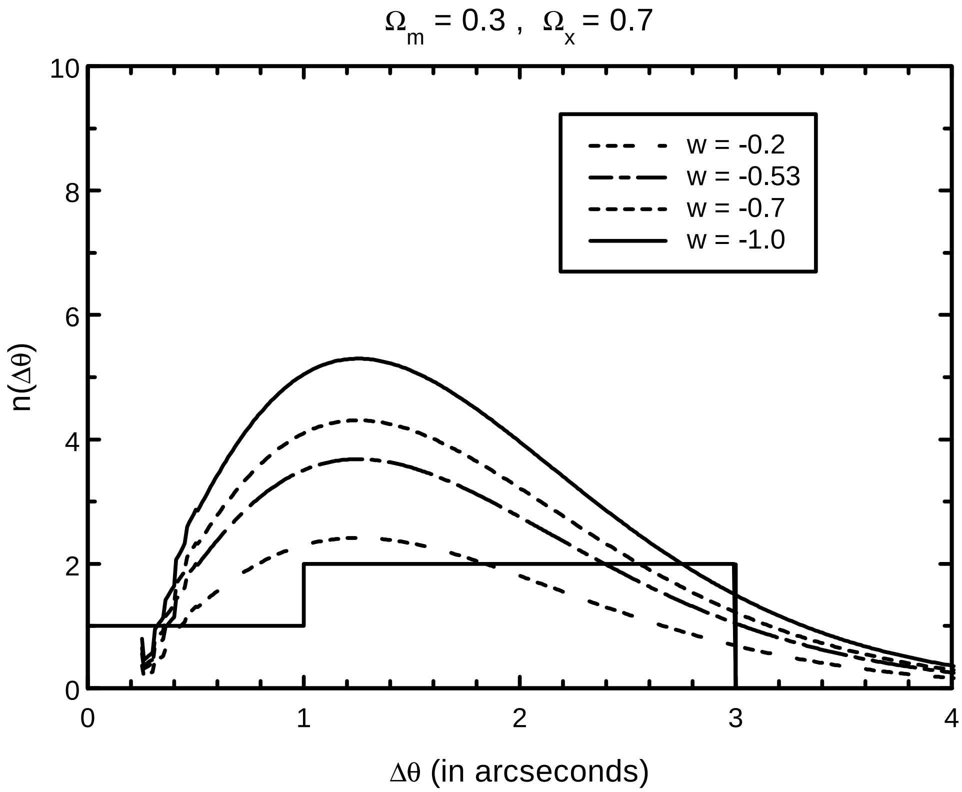

Figure 2: The expected distribution of lens image separations with $\Omega_m = 0.3, \Omega_x = 0.7$ compared with the observed image-separation distribution in the optical sample (histogram). The expected distribution is plotted for different values of w.

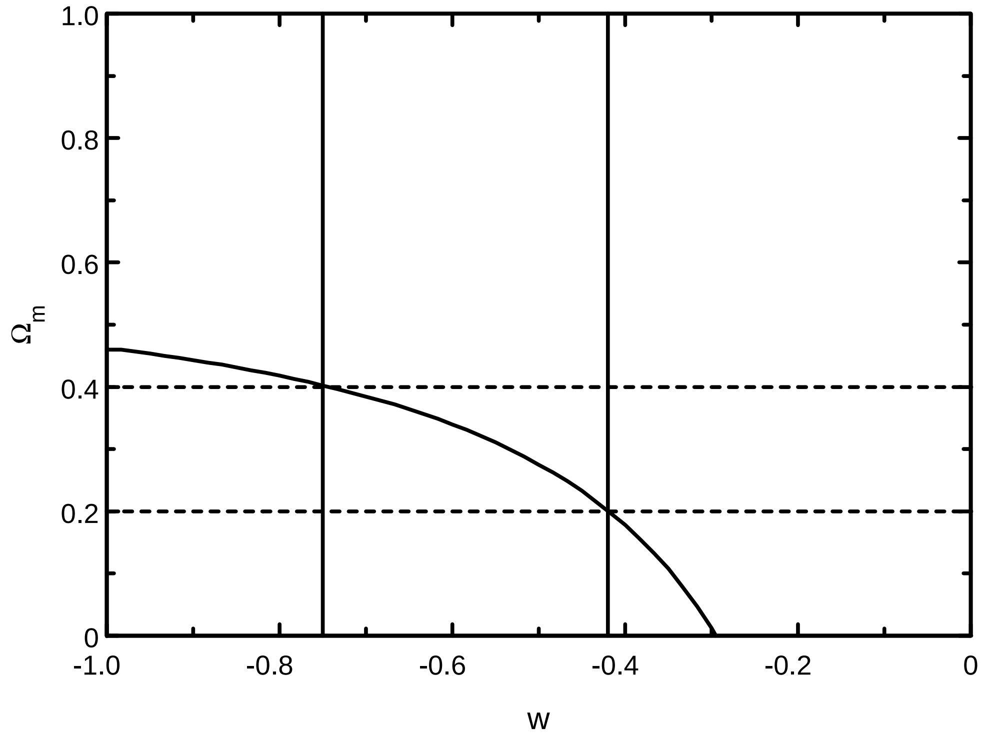

Figure 3: Contour between Ω_m and w in order to get five lensed quasars in the optical sample. The dotted lines indicate the observed range of $\Omega_m \sim 0.2 - 0.4$. The vertical lines give the corresponding value of w.